An Approach to Include Tonnetze in School's Music Class Curricula and Why This Can Be a Good Idea

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Abstract

Neo-Riemannian Theory and especially the visualization of its key principles in the form of the so-called “Tonnetz” figure is widely perceived as one of the main foundations for analyzing music. Most prominently, with respect to the epoch of Romanticism, Tonnetze can facilitate understanding the basic principles of harmonization and offer a visualization of the transition functions between triads. On a more mundane level, Tonnetze—with their advantageous visualization—can be beneficial for introducing the key principles of triads with respect to major and minor keys. For all these reasons, this paper aims to give an idea of how to use the knowledge that Neo-Riemannian Theory provides and Tonnetze as one specific representation thereof in music education for children and adolescents of varying competence levels. In this paper, we will, thus, give a brief introduction to Neo-Riemannian Theory and Tonnetze before providing suggestions—supplemented by adequate material—on how to include Tonnetze and Neo-Riemannian Theory in the curriculum of different educational levels.

1 Introduction

Neo-Riemannian Theory (NRT) is a segment of Transformational Theory (a subbranch of music theory that joins music and mathematics) dealing only with major and minor triads and consisting of three basic contextual inversions. It is based on an idea of the German music theorists Hugo Riemann and David Lewin to look at intervals in a mathematical way. Such a view gives the freedom to analyse music independently from the key and to find new chord progressions by focusing on the relations between triads. They mark an important element in the expansion from the basic music theory for harmony—the Functional Theory—in the Classical epoch to more diverse chord transitions in the epoch of Romanticism (Cohn, 1998).

To make progressions over the scale more smooth and still ensure harmony, NRT tries to have as little change as possible between two consecutive chords. This is achieved by three types of small changes, the so-called P-, R- and L-transitions. As all triads can be put into relation to each other by a sequence of these transitions, NRT can be visualized in so-called “Tonnetze”, which can be portrayed differently depending on intended usage (Andreatta & Baroin, 2016).

The easy harmonic transitions from minor to major as well as the possibilities for music analysis and visualisation render NRT and Tonnetze very helpful fundamentals in the music class curricula. They allow teaching personnel to address all learning types and to address students with differing prior musical knowledge by simply adapting the Tonnetze.

Though the relation to the subject of Cognitive Science seems flimsy at first glance, it is not to be disregarded that proper teaching takes knowledge from various disciplines, e.g. (developmental) psychology, into consideration. Ideally, teaching is not only concerned with putting concepts into descriptive words, but with offering analogies and explanations that optimally target the student’s current knowledge and developmental stage.

The paper begins with a brief introduction to NRT and Tonnetze before giving reasons for including Tonnetze into the curriculum. Afterwards, specific ideas for where and how to include the Tonnetze in the curriculum are provided, as well as guiding materials (see Appendix).

2 Neo-Riemannian Theory and its Tonnetze explained in more detail

In the following section, Neo-Riemannian Theory, as well as a representation thereof—the Tonnetze—shall be introduced in detail.
2.1 Neo-Riemannian Theory

The main concept of NRT is to look at intervals in a mathematical way and, therefore, to abandon the importance of an overall key for a composition such as in functional harmony (Gollin, 2005). It is based on three conceptual inversions—the P (parallel), R (relative) and L (leading-tone) transition:

- A parallel transformation (P) converts a major triad into a minor triad and vice versa by moving the third by a half-step (e.g. C major to C minor) (see Figure 1).

  ![Figure 1: A parallel transformation.](image1)

- A relative transformation (R) converts a major triad to a minor by moving the fifth a whole-step up and moves a minor triad to a major by moving the root down by a whole step (e.g. C major to A minor) (see Figure 2).

  ![Figure 2: A relative transformation.](image2)

- The leading-tone transformation (L) converts a major triad to a minor by moving the root down by a half-step and changes a minor triad to a major by moving the fifth up a half-step to become the root of the resulting triad (e.g. C major to E minor) (see Figure 3).

  ![Figure 3: A leading-tone transformation.](image3)

Each transformation is an involution, also called an inverse, which alters the mode from major to minor or vice versa and preserves two common tones and changes the third by a semitone (in P and L) or a whole tone (in R). Having such little change between two triads is also known as the voice-parsimony in which the goal is to smoothly move from one set of harmony to the next.

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From these basic transitions (see Figure 4), music can be easily analyzed with NRT. Since each triad can now be connected to three others and each of them respectively to two new ones (the third being the involution), every triad can be now connected to all the others by using more than one transition at a time. When having more than one transition, it is spoken of “compound transitions”. They are labelled with the order in which the transitions were applied. So if one wants to label the chord progression from C major to A major, it would be a “RP” compound transition since we can transition from C major to A minor with a relative transition and from A minor to A major with a parallel transition. In contrast to a single transition, a compound transition is not an inversion. To go back from A major to C major one would need to do a “PR” transition (see Figure 5).

It can be concluded that adding NRT to Functional Theory gives more options when analyzing music, since instead of the seven triads linked to a certain key, all major and minor triads are connected. Especially in late Romanticism, this was very helpful since composers focused more on expressing emotion than having good harmony (Mason, 2013).

2.2 Tonnetze

There are various ways as to how the knowledge that NRT provides us with can be visualized, one of which is the Tonnetz figure, which, in turn, can result in a variety of different realizations itself. In this introduction to Tonnetze, we will mostly focus on a representation that comprises of only the tones on the chromatic scale as it is shown in Figure 6.

This Tonnetz comprises all tones of the chromatic scale (C to B) including all the semitones in between (for example C#, the tone between C and D), yet it is not the tones themselves that enable the Tonnetz to be extraordinarily beneficial to the application of music theory—it is the particular way of ordering them.

Figure 4: Summary of all transformations, the first of which is a parallel transformation, the second a relative, and the last one a leading-tone transformation. Adapted with added chord names from “Essential Neo-Riemannian Theory for Today’s Musician” (Master’s Thesis), by L. F. Mason, 2013, University of Tennessee, Tennessee Research and Creative Exchange (TRACE), p. 7 (https://trace.tennessee.edu/utk_gradthes/1646/). Copyright 2013 by Laura Felicity Mason.

Figure 5: An illustration of a compound transition consisting of a relative and a parallel transition. Image created for this paper. Copyright 2023 by Laura Hellmann and Melissa Jansen.
In a two-dimensional space, the tones are arranged in a network (in German called “Netz”, Tonnetz) that is crucial for the correctness of its contained information. Horizontally, the tones are arranged in an order that adheres to the rule of the perfect fifth, which means that there are seven semitones between two adjacent tones—F, followed by C, which is followed by G. In the vertical dimension, the tones are arranged in chromatic order which means that adjacent tones have a distance of one semitone—C, followed by C, followed by C#. Lastly, there are the diagonals, the ordering of which is imposed by the ordering within the horizontal and vertical dimension, but which, nevertheless, shows a regular pattern. This pattern adheres to the rule of major and minor thirds (4 or 3 semitones of distance, respectively), depending on which direction is taken. When going from top-left to bottom-right, this is the direction of the major thirds which usually sound happy and upbeat—A is followed here by C, which is then followed by E. In the other direction, from top-right to bottom-left, the thirds are minor thirds, usually creating rather melancholic emotions in the listener, where E is followed by C and then by A.

There are various ways as to which tones are included in the Tonnetz and which particular ordering is applied to determine the functionality of the Tonnetz. In the example that is referred to in the scope of this paper—the most common Tonnetz that there is—the arrangement of tones enables the characterization of triads just by the spatial composition of the tones in the two-dimensional space. The three characteristics that are crucial in this representation of the Tonnetz are (1) color, (2) orientation, and (3) the tone at the corner facing leftwards. For the example of an A minor chord (C-A-E), it can be seen that the three tones are located at the corners of a blue triangle with the tip facing upwards, where A is in the lower left corner. In contrast to that, the C major chord (C-G-E) encapsulates a red triangle that is oriented downwards and that has the tone C at the left corner. These two illustrative examples allow for the conclusion that, in order to determine the name of the triad based on three separate tones, the type of chord—whether it is major or minor—is clearly determined by the color and orientation of the triangle (red and downwards for major; blue and upwards for minor), whereas the tone at the left corner clearly identifies the name of the chord. With this procedure, it is possible to specify the name of a triad on the basis of three tones, as well as determining the tones a specific triad is composed of. Another beneficial feature of this Tonnetz
is the fact that it has a superimposed color gradient highlighting chords that are more common in the centre as opposed to chords that include a lot of accidentals in the periphery.

For the sake of completeness, we find it important to note the fact that some representations of Tonnetze enable the determination of not only triads, but also more sophisticated chords, for example dominant or diminished chords, as well as major and minor 6th or 7th chords. These, too, can be characterized by shape and position of a particular tone determining the name of the chord (Bergstrom et al., 2007; Cannas et al., 2017).

In and of itself, this function of the Tonnetz can prove to be very handy, yet it is not the only functionality it offers. Apart from defining chords on the basis of the tones that they consist of, these chords have a special positioning relative to each other in the Tonnetz as well—they encapsulate the transition functions of the leading-tone, relative, and parallel transition between chords. As every Tonnetz can be structured differently, it can happen that a leading-tone transition is represented by horizontally shifting from the center of a triad to the right in one Tonnetz, while it is vertically going down in another. A representation that is very beneficial for representing these relations is depicted in Figure 7.


Here, the leading-tone transition is always happening on the vertical axis, for example between C major (C-G-E) and E minor (E-G-B). A relative transition, as between C major (C-G-E) and A minor (C-A-E), is realized by an ascending shift from left to right, whereas descending from left to right provides the reader of the Tonnetz with the respective parallel triad. Following the previous examples, when performing a parallel transition on the tones forming C major (C-G-E), it yields C, E₆, and G, which is C minor.

It would be convenient to stop here and appreciate the order of this visualization that manages to encapsulate a large amount of knowledge, but there is still another level of sense behind this structure that becomes evident when listening to the triads instead of only seeing them visually displayed, since the fact that two adjacent triads always match in two tones did not emerge arbitrarily. As much as this principle of switching only one tone adheres to the Neo-Riemannian transition rules of leading-
tone, parallel, and relative transition, it is widely understood that two triads that are connected via such a transition also sound pleasurable together. This way, proximity or distance in the Tonnetz does not only account for structural similarity, but also for tonal suitability, which yields a practical tool for creating fitting chord progressions without the need to deeply study music theory.

3 Why include Tonnetze in the curriculum?

For the exact reason that Tonnetze visually compress and simplify tremendous amounts of musical theory without requiring the user to bring a lot of knowledge, we suppose that Tonnetze and the various interactive applications there are, can be beneficial supplementary material in everyday music lessons at varying levels of education. Since there are the different levels of structural coherence within the Tonnetz—determining triads, characterizing transitions, and knowledge about harmonization—we deem it suitable for both introducing children to music and analyzing music from the epoch of Romanticism, as well as the abundant possibilities in between. Furthermore, we think that Tonnetze can be used to link topics throughout the course of study, achieve low-level introductions to new topics by the simplified layout of the Tonnetz, provide students with strengths in visual learning with appropriate learning material, and tighten knowledge that has been transferred via a different medium by repetition.

In order to bring theory to the practical application and put forward suggestions as to how Tonnetze could be included in the curriculum, we looked into the syllabuses of different German federal states, provided that we found material ("Kernlehrpläne", n.d.; "Musik", n.d.). It soon struck us that curricula differ immensely between the federal states, let alone the differences we encountered when comparing the syllabuses of different school forms or deviating national school systems (Kertz-Welzel, 2004). As a consequence, we tried not to generalize on the institutional, but rather on the developmental level, by applying our suggestions to the different competence levels that are necessary for music education. We give examples of which topics can be understandable at that very specific stage of development and how Tonnetze can be a beneficial supplementary—or maybe even central—part of the learning process.

4 Ideas for including Tonnetze

In the subsequent section, the theoretical basis that was presented beforehand shall be applied by coming up with suggestions for teachers working with students of diverse age groups.

4.1 First encounters with music

When getting in contact with music for the first time, the Tonnetz, or at least an operable representation of it, can be beneficial. The advantage of a Tonnetz representation, for example the one made accessible by IMAGINARY (The Tonnetz - One Key, Many Representations, n.d.), is that chords can be played without necessarily being able to play an instrument or having any music theoretical knowledge. For children at a very early age or anyone who has not yet come into contact with music in a self-determined manner, this can be the perfect medium (Bigo et al., 2012; Guichaoua et al., 2021).

For the same reason that the Tonnetz is so advantageous, it is difficult to set topic-specific learning goals for this stage. This stage is supposed to be about self-discovery, trying out music, listening, hearing what sounds good and what does not, and thereby acquiring a sense of agency over sounds (Jaffurs, 2004).
Due to the fact that this learning objective is formulated with a lot of freedom included, possible tasks as well as the medium in use must allow for such creative independence as well. When working with groups that have a strong sense of self-determination and that are able to focus, a suitable task can be to let them experiment on their own with an operable representation of a Tonnetz just as described above. Groups that need a bit more guidance could be given the task to find regularities or irregularities between triads that are close-by or far distant from each other in the Tonnetz. Since distance in the Tonnetz is related to less tonal harmony, they make the same experiences as the more independent group who independently considers which triads sound good in combination and which do not.

4.2 About minor and major: introducing triads

As has already been mentioned in the introduction, a major feature of Tonnetze is the fact that they visualize all triads in an easy and structured way. As a result, it is straightforward to determine the name of a triad from its constituting tones and vice versa. Giving an introduction on what the Tonnetz is built of and how the tones are structured to constitute this function is sufficient to let students effectively work with the Tonnetz.

When it comes to mastering the subject of triads, the most important learning goal is to understand how triads are constructed. In the end, they should be able to explain the structural difference between a major and a minor triad and how they can determine the name of a triad when being given the tones and vice versa. Taking a step further, another learning goal could be to understand the relationship between major and minor, since the parallel transition is easily spotted in the Tonnetz and the visualization yields a decent foundation to understand deeper concepts. Thus, even within this stage, it is possible to include different levels of difficulty.

In order to train this in an applied manner, one option would be to give the students several chords in the form of the constituting tones or in the form of the chord names and let them determine either the name or the tones with the help of the Tonnetz. They would acquire not only the skill of navigating the Tonnetz, but might also detect regularities and rules within these examples—for example, if they are asked not only to write down the tones as letters, but mark them correctly in a music sheet. A way to make the students interact deeper with the Tonnetz would be to let them translate the Tonnetz from sharp to flat. An important aspect that would be transferred is the interchangeability of names for triads, since the students would realize that E\# major and D\# major are the same triad, even though they might look different.

4.3 Gaining an overview of chords and their structural relationships

When looking at a Tonnetz, it is easy to understand the difference between major and minor triads, as only one note and respectively one corner of the chord triangle is different. It is, furthermore, easy to determine which transition or sequence of transitions takes place between triads.

Additionally, Tonnetze can be well introduced together with the Circle of Fifths (Figure 8) from Functional Theory. Instead of focusing on classical progressions like moving from a Dominant to a Tonic, students can learn the basic transitions of the NRT to already have a grasp on all these pieces for future analysis of music sheets or just for having more options when composing. Linking Tonnetze and the Circle of Fifths can be especially beneficial for students learning visually. When including the basic transitions from the NRT into the Circle of Fifths, especially the parallel and leading-tone transition can be visualized quite nicely and might be easier to remember for students who do not fancy reading notes or doing inversions of triads. Similar to every right neighbor in one mode (major or minor) being a Dominant and every left one being a Subdominant, the relative transition is the direct neighbor in the other mode (C major and A minor or E\# major and C minor).
and the leading-tone transition is the diagonally to the right neighbor in major mode or diagonally left neighbor in the minor mode.

Figure 8: Circle of Fifths illustrated with transitions. Image created for this paper. Copyright 2023 by Laura Hellmann and Melissa Jansen.

Sadly, the parallel transition does not work nicely in this environment, but since this transition is the most intuitive, coloring just one example could be sufficient.

4.4 Little composers

By now, we mostly discussed Tonnetz applications in which the Tonnetz is either just a visualization of knowledge that has been acquired in a different manner or a useful tool to extract information from given pieces of music. As soon as the functionalities and features of the Tonnetz have been understood, it becomes increasingly operable for compositional purposes instead of the passive role it has previously taken. With the help of a Tonnetz and the relations that are defined, it is possible to build a harmonic base in little time and even by trial-and-error, if necessary. Simultaneously, the Tonnetz offers a lot of creative freedom, as it is possible to not only stick to the basic transitions, but also to come up with a sequence of these transitions—compound transitions—to travel the Tonnetz a bit further and include more variability.

The goal of this last stage shall thus be to use the Tonnetz, as well as all the previously acquired knowledge, to compose something that can either be entirely personal or a part of an existing piece. Given the objective to get to know the Tonnetz in a creative instead of a restrictive setting, it is advantageous that students experience the process of music creation which makes it more accessible and which can also have self-esteem benefits (Jaffurs, 2004; Kertz-Welzel, 2004).
A possible task could be, of course, to leave the students with an introduction on how to make use of the Tonnetz and allow them to compose their own chord progression or maybe even an entire piece. Students who require more guidance could be given a theme to compose a basic harmony, e.g. something that sounds upbeat, something that sounds eerie, or something that sounds epic. To have an even more applied task framing, they could be given a situational description of a movie scene which should be embellished with the help of ambient music. Nevertheless, it is important to keep in mind that what sounds good and what does not, likewise what fits and what does not, is highly subjective and strongly tied to personal preference that is rooted in a person’s character. In schools, it can be that even though everyone knows that tastes diverge, any deviations from the norm or the average are not appreciated and ill-treated by the class or course community. Students might feel discomfort in putting out their own preferences or could feel pressured to come up with something extraordinary to fulfill the task. To relieve them of this pressure, the students might be given a short video sequence with which their music should be co-played in order to shift the focus from judging the music itself to judging whether it fits the film sequence which is less subjective.

4.5 Inspecting the beauty of Romanticism

Following the Classical epoch, the epoch of Romanticism emerged. In this epoch, composers still used classical formalism, such as Functional Theory, but did not feel confined to this. Following the assumption that music is the best way to convey human emotion, the focus was no longer on harmony. Dissonance and unknown chord progressions were included in compositions and changed the music significantly.

Using the NRT and its Tonnetze as tools for analyzing music next to the basic progressions such as Tonic, Dominant and Subdominant (maybe also double Dominants depending on prior knowledge), analyzing compositions gets a lot more interesting. Students have the option of using the Tonnetz, the Circle of Fifths or just their theoretical knowledge depending on their learning preferences when analyzing music. Also finding a scheme such as Hamiltonian cycles—having a sequence of transitions leading back to the starting chord—might be more exciting than not being able to explain chord progressions not fitting to the compositions’ key. In class, comparing Classical compositions to Romantic ones can give the basic idea of differences between the two epochs and gives opportunity to also let students figure out from which epoch a composition seems to be. Just analyzing music is the most intuitive way of including the NRT and its Tonnetze into the music class curriculum at this level.

5 Conclusion

To summarize, we took a close look at Tonnetze in their position of being useful visualizations of NRT. Since there are different levels of abstraction within the Tonnetz, we think that it is a suitable teaching medium for schools’ music classes—be it as a guide, as the main material, or as a follow-up. From our research about common themes in schools’ curricula, we realized that the variance of topics for the different age groups is too large to generalize with respect to our suggestions. To make up for that, we opted for a thread in which every stage of music education can be part of a school’s music education curriculum and in which every possible reader can position themselves at a certain point with respect to their level of competence. For each of these stages, we attempted to specify the features that the Tonnetz offers for this specific level, as well as the learning objectives we deem suitable. To offer examples of how to put this into practice, every section contains at least one task formulation that shall help to work towards or to achieve this goal.

Of course, it remains to be seen in how far these task suggestions and learning objectives are not
only reasonable, but also realizable. However, these task descriptions are purposefully kept rather
general, such that adjustments can be made to tailor them to the group at hand, which is one
of the key criteria to successful teaching. Even if Tonnetze happen to serve only as a secondary
medium to strengthen the knowledge that was taught differently, it can be a beneficial addition to
the curriculum to add more variance to the material and make the information accessible to various
learning types. Giving students abundant learning opportunities without overwhelming them and
letting them try out what fits their needs and abilities best is a way in which schools can support
their children and adolescents on a long-term basis other than teaching bare material and we think
that including Tonnetze in school’s music curricula is one option to put this into practice.
References


6 Appendix

Beethoven’s Ninth Symphony (even though historians argue about his works already counting as compositions of the epoch of Romanticism) has an excerpt only consisting of repeating R and L transitions, which gives a great example for practical implementation of very simple NRT transitions by a popular composer. Using this excerpt (Figures 9 and 10) one can ask the students to look for the transitions or maybe let them compose the last four measures on their own. Also one can give them a reduction of this and ask them to either identify the chords or find the transitions. The reduction would be:

\[C \rightarrow Am \rightarrow F \rightarrow Dm \rightarrow B \rightarrow Gm \rightarrow E_b \rightarrow Cm \rightarrow A_b \rightarrow Fm \rightarrow D_b \rightarrow B_b m \rightarrow G_b \rightarrow E_b m \rightarrow C_b \rightarrow A_b m \rightarrow E \rightarrow C_\# m \rightarrow A\]

Figure 9: Beethoven’s Ninth Symphony, mm. 143-175, without chord names. Reprinted from "Essential Neo-Riemannian Theory for Today’s Musician" (Master’s Thesis), by L. F. Mason, 2013, University of Tennessee, *Tennessee Research and Creative Exchange (TRACE)*, p. 48 (https://trace.tennessee.edu/utk_gradthes/1646/). In the public domain.
Figure 10: Beethoven’s Ninth Symphony, mm. 143-175, with added chord names. Adapted from “Essential Neo-Riemannian Theory for Today’s Musician” (Master’s Thesis), by L. F. Mason, 2013, University of Tennessee, Tennessee Research and Creative Exchange (TRACE), p. 48 (https://trace.tennessee.edu/utk_gradthes/1646/). In the public domain.
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